INCOSE WMA Tutorial on Prescriptive Analytics

George Mason University

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Decision Theory in Data Analytics

Where Innovation Is Tradition

Agenda

- Decision Theory and Prescriptive Analytics
- Deciding with Values
- Qualitative and Quantitative Assessment
- Modeling Uncertainty
- Risk Analysis

What makes a decision hard to make?

- Competing objectives
- Uncertainty
- Differing perspectives of Decision makers
- Disagreement over what is to be accomplished
- Politics
- Lack of information
- Complexity

What is a Decision?

- A decision is an irrevocable commitment of resources
  - Money
  - Time
  - People
- A decision **must** have
  - Alternatives to choose among
  - Outcomes for each alternative
  - A value structure within which to order / rank outcomes
  - Assumptions relating to all of the above
- A decision **may** have
  - A probability measure on possible outcomes
  - A probability measure on future conditions (states of nature)
  - An active opponent trying to defeat your decision

The Elements of a Decision

- Objectives and means
- Alternatives to choose between
- Uncertainty in Events and Outcomes
- Consequences of the Decision

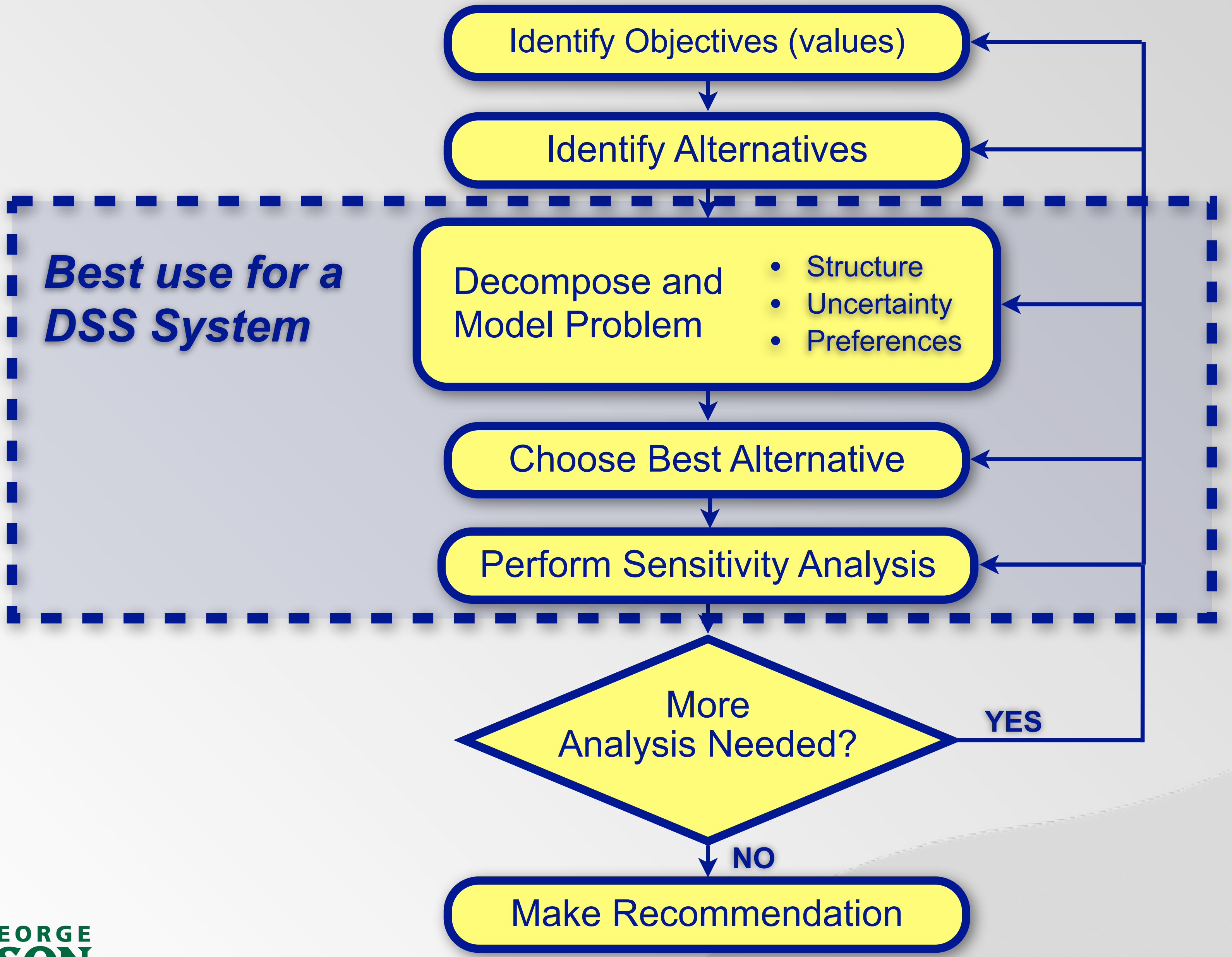
What is Decision Analysis?

- A Method to:
  - Organize or structure complex problems for analysis
  - Deal with tradeoffs between multiple objectives
  - Identify and quantify sources of uncertainty
  - Incorporate subjective judgments

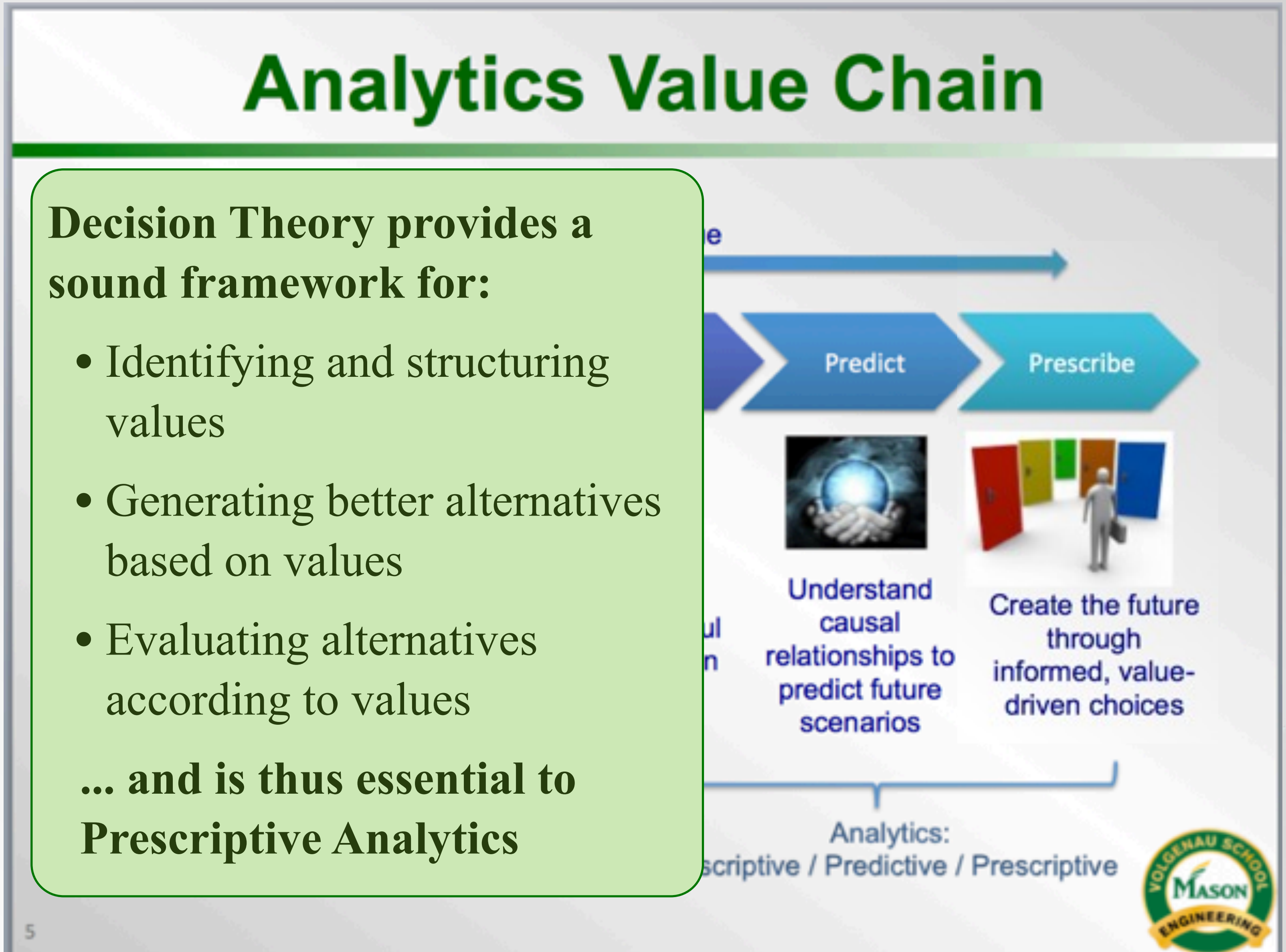
Decision Analysis Tenets

- Quality decision making requires a *systematic process* to incorporate
  - Information, expert opinion, and preferences
- Complex decisions in large organizations involve
  - *Functional experts* (inside)
    - R&D, engineers, operations, production, finance, etc.
  - *Interested stakeholders* (outside)
    - Stockholders, government, community, etc.
- *Quantification* offers significant benefits
  - Clarifies thinking
  - Values
  - Uncertainties (Probability)
  - Consequences
  - Improves communications
  - Enables logical reasoning
- 4. *Support decision maker* judgments by providing *insights* (more about dealing with decision makers later)

Prescriptive Analytics Process



Where Decision Theory Helps the Most?



Decision Theory provides a sound framework for:

- Identifying and structuring values
- Generating better alternatives based on values
- Evaluating alternatives according to values

... and is thus essential to Prescriptive Analytics

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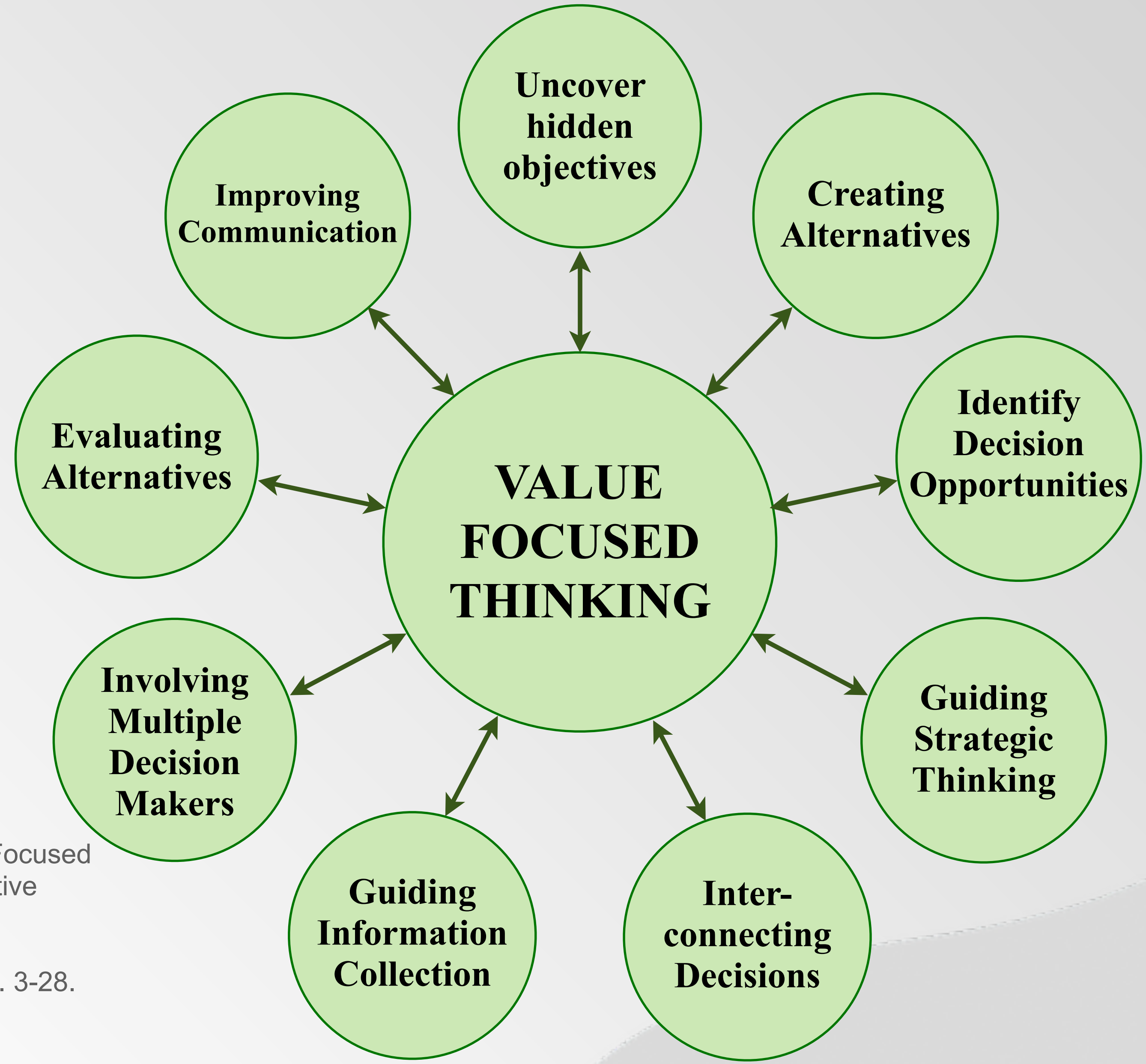
Value Focused Thinking

- Naïve decision making is alternative-focused
  1. Problem arises
  2. Begin problem solving
  3. List alternatives
  4. Choose alternatives
- Why is this a limited way to think?
  - Reactive, not proactive
  - Encourages incomplete analysis
  - Focuses on status quo and a few minor variations
- Value focused thinking starts with decision-maker's values
  - Identify and structure values
  - Generate better alternatives based on values
  - Evaluate alternatives according to values

Where does thinking about values lead?



Keeney, Ralph L., Value-Focused Thinking: A Path To Creative Decisionmaking, Harvard University Press, Cambridge, MA, 1992, pp. 3-28.

Value Modeling

- Facts, values, objectives
  - Facts - objectively verifiable pieces of information about events or circumstances in the world
  - Values – how a stakeholder feels about the events or circumstances
  - Objectives – what a stakeholder is trying to achieve
- Decision maker's values should drive decision
- Objectives should be consistent with values
- Value model
  - Explicitly represents benefits and costs of each decision alternative
  - Supports rational comparison of alternatives
  - Usually build qualitative value model first and move to quantitative model as required
  - Helps overcome things that make decision difficult

Identifying Values

- Interact with decision makers
- Elicit feedback throughout DSS development
- Values are stated through **objectives**
- 3 Features of objectives
  - (1) Decision Context
  - (2) Object
  - (3) Direction of Preference
- Ex: Forest products company wants to "minimize environmental impact"
  - (1) Decision Context - ***Harvesting Trees***
  - (2) Object - ***Environmental Impact***
  - (3) Direction of Preference - ***Minimum is best***

Ways to Identify Values

(due to Professor Gregory Parnell, USMA)

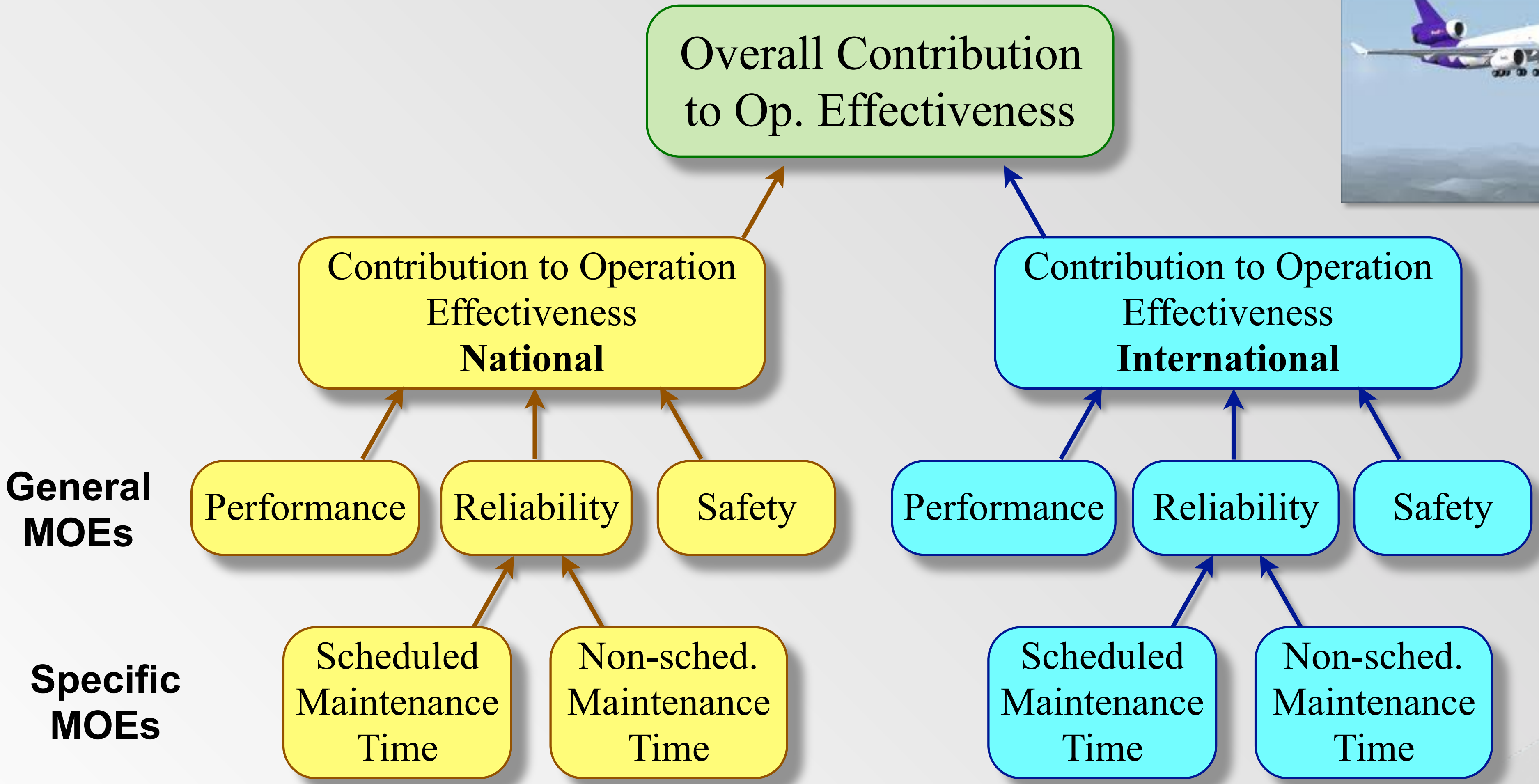
- **Gold Standard**: based on an approved vision, policy, strategy, planning, or doctrine document
  - Values have been thought about, discussed, and written down
  - Use work that has already been done and approved
- **Platinum Standard**: based on interviews with decision-makers and stakeholders
  - Often difficult to get enough time with stakeholders and DMs
  - Use Affinity Diagram for group settings
- **Silver Standard**: uses data provided by stakeholder representatives
  - When real DMs and stakeholders are not available
  - Still use Affinity Diagrams for groups
- Combined Standard: combination of the above

Structuring Objectives

- Initial list has “non-objectives”
  - Alternatives
  - Constraints
  - Criteria for evaluation
- Convert “non-objectives” into objectives
- Types of objective
  - ***Strategic objectives*** - objectives that describe a person’s life or an organization’s purpose
  - ***Fundamental objectives*** -- essential to decision maker’s concerns
  - ***Means objectives*** -- important only because of relationship to fundamental objectives

Strategic objectives are stable over time although the means to obtain these objectives change with time and context.

Value Hierarchy for Logistic Operations



- Complete
- Consistent
- Measurable
- Non-overlapping
- Hierarchically organized
- 7 or fewer per group

Generating Alternatives

- No recipe – but "Values First!"
- Consider obvious alternatives (including status quo)
- Look for alternatives that satisfy subset of objectives (2 or 3 at a time)
- Try to modify alternatives to overcome shortcomings
- Combine several alternatives into a new alternative
- Focus on strategic objectives
  - Most important
  - Broader in scope
  - Will do most good

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Qualitative Assessment

- Build a qualitative value structure and generate a list of alternatives
- This process helps to:
  - Clarify thinking of all participants
  - Improve communication among stakeholders
  - Identify the most important objectives
  - Specify ways to measure how well objectives are satisfied
- Eliminate obviously inferior alternatives
  - If an alternative is dominated (scores worse than another alternative on all attributes) we can eliminate it
  - We may be able to eliminate some non-dominated alternatives if they are clearly worse than other alternatives
- If several alternatives still remain, we need build to quantify tradeoffs among competing objectives

Quantitative Value Function

- a.k.a. “Objective Function” or “Multi-attribute value function”
- Denoted v(x)
- Assigns a number to the consequences, x, of an alternative
- Used to determine preference among alternatives
- Types of value functions
  - Ordinal value function - ranking only
  - Measurable value function – measures strength of preference
  - Utility function – measures risk attitude
- Form depends on relationships between attributes
  - Attributes are quantitative measures of how well a consequence satisfies an objective
- Convention: $v(x^A) > v(x^B)$ iff we prefer alt A to alt B

Structures of Value Functions

Additive Value Function

$$v(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} k_i v_i(x_i)$$

where k_i is a positive scaling constant
v and v_i are value functions scaled from 0 to 1

All combinations of attributes are Preferentially Independent!

Structures of Value Functions

- Multiplicative Value Functions

$$1 + kv(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} [1 + kk_i v_i(x_i)]$$

All pairs of attributes are Weak-Difference Independent

– Example:

$$v(x_1, x_2) = k_1 v_1(x_1) + k_2 v_2(x_2) + k_3 v_1(x_1) v_2(x_2)$$

– Looks like regression equation with interactions
– More difficult to use

Bottom Line: Use Additive Independent attributes!

Example (1 of 4)

- Problem Statement: What new aircraft system should Fedex buy to improve the effectiveness of its logistic operations?
- Procedure:

  (0) Choose <u>Fundamental Objective</u>

    – Best mix of safety, reliability, and performance

  (1) Determine important <u>Means Objectives</u> (multiple scenarios)

    – Contribution to safety

    – Contribution to reliability

    – Contribution to operational performance

Example (2 of 4)

(2) Find measures for objectives

- Safety: Events per 100K hours, Accidents per 100K hours
- Reliability: Maintenance downtime, Non-Scheduled downtime
- Operational Performance: Tons per flight, Average Ground Time

Note: Measures will be computed separately for each scenario of interest.

Example (3 of 4)

(3) Scale measures

- Convert each objective to scale from 0 to 1
  - 0 means “reasonable worst” score
  - 1 means “reasonable best” score
  - others measured proportionally
- Simplest transformation is linear:

$$v_{attrib}(x) = \frac{x - worst_{attrib}}{best_{attrib} - worst_{attrib}}$$

- Limitations:
  - Ignores riskiness of alternatives
  - Assumes increment is valued equally at any point on scale from worst to best
  - Can define more sophisticated rescaling functions

Example (4 of 4)

(4) Weight the objectives (criteria)

Answer the question: How do we trade increased value in one objective for lower value on the others?

– Methods:

- Trade-off
- Smarter
- Swing weights *
- Lottery weights

– Often totally subjective

– Avoid "Importance Weights"

– Must consider range of variation of attributes

Elicitation Techniques

- Swing Weighting
  - Emphasizes the Range of Variation of variables
- Trade-Offs
  - A Derivative of Swing Weighting in Logical Decisions
- Simple Multi Attribute Rating Technique (SMART&SMARTER)
  - An Ordinal Technique
- AHP

Swing Weights

- Based on comparing ranges of variation of attributes
- Can be used for non-quantitative attributes
- Method:
  - Find "Worst Conceivable Alternative"
    - Lowest score on each attribute
    - May be imaginary
  - Pick attribute that gives greatest improvement when "swings" to highest level
  - Pick attribute that gives next highest increase when swung to highest level
    - by percentage - how does it compare with the first?
    - never greater than 1 since first is best
  - Repeat for rest of attributes
  - Solve for weights

$$k_2 = k_{21} w_1$$
$$k_3 = k_{31} w_1$$
$$\vdots$$
$$k_n = k_{n1} w_1$$
$$\sum_{i=1}^{n} k_i = 1$$

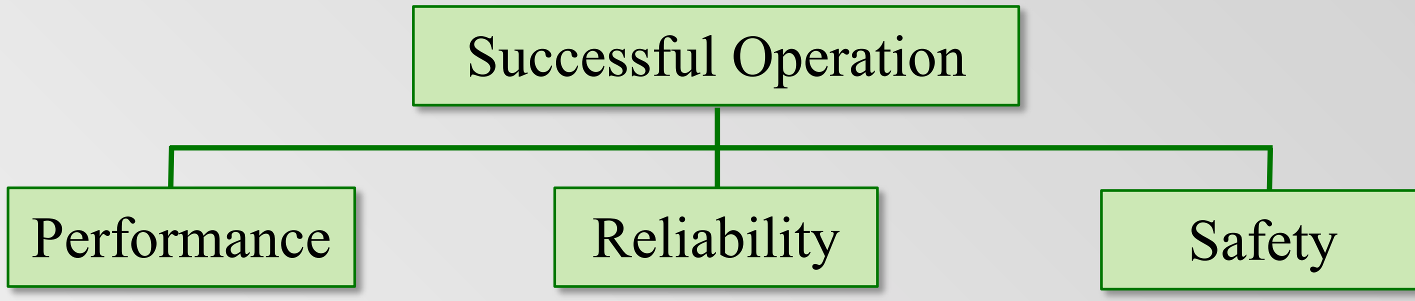
- k_i is weight of i^{th} attribute
- p_{i1} is percentage improvement compared with attribute 1

Swing Weights in a Value Hierarchy

- Each level has its own additive value function with weights
  - Weights sum to 1
  - Weights correspond to relative importance of swings on each of the criteria
  - Weights assessed as in any additive value function
- Each additive value function is independent of the others

http://c4i.gmu.edu/~pcosta/files/INCOSE_DTexample.xlsx

Example: Swing Weight Elicitation



	Performance	Reliability	Safety
BEST	.95	27 days/month	.52 A/100Kh
	.125 3 .25	.375 2 .75	.500 1
WORST	.92	25 days/month	.80 A/100Kh

$$K_{cost} = (1.00) * K_{cost} = 0.5$$

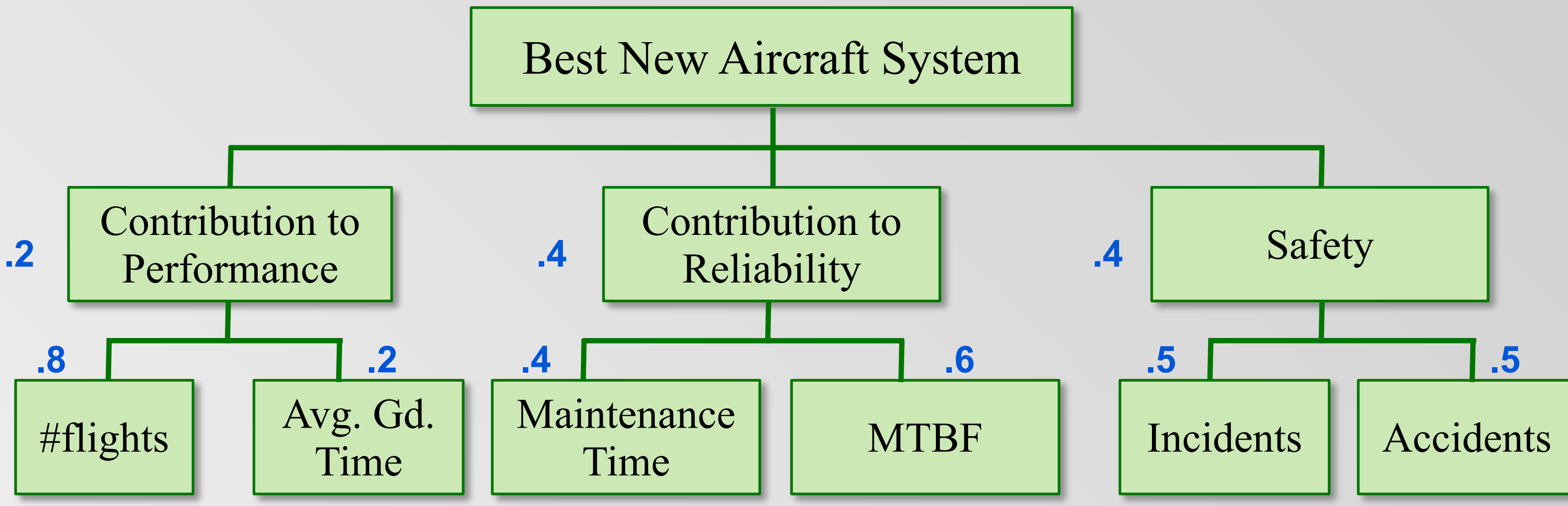
$$K_{time} = (0.75) * K_{cost} = (0.75)*(0.5) = 0.375$$

$$K_{perf} = (0.25) * K_{cost} = (0.25)*(0.5) = 0.125$$

$$1 = 2.0 * K_{cost} \longrightarrow K_{cost} = 0.5$$

Example Problem – Choose the Best Aircraft

Value Hierarchy With Weights



Bottom Row Weights .16 .04 .16 .24 .20 .20

Simulation Results

Alternative Aircraft System	# flights per Op. Month	Avg. Ground Time (m)	Maintenance Time (days)	MTBF (days)	# of Incidents /100Kh	# of Accidents / 100Kh
1	385	26	8	85	22	0.52
2	420	32	4	92	28	0.73
3	350	38	10	120	18	0.80

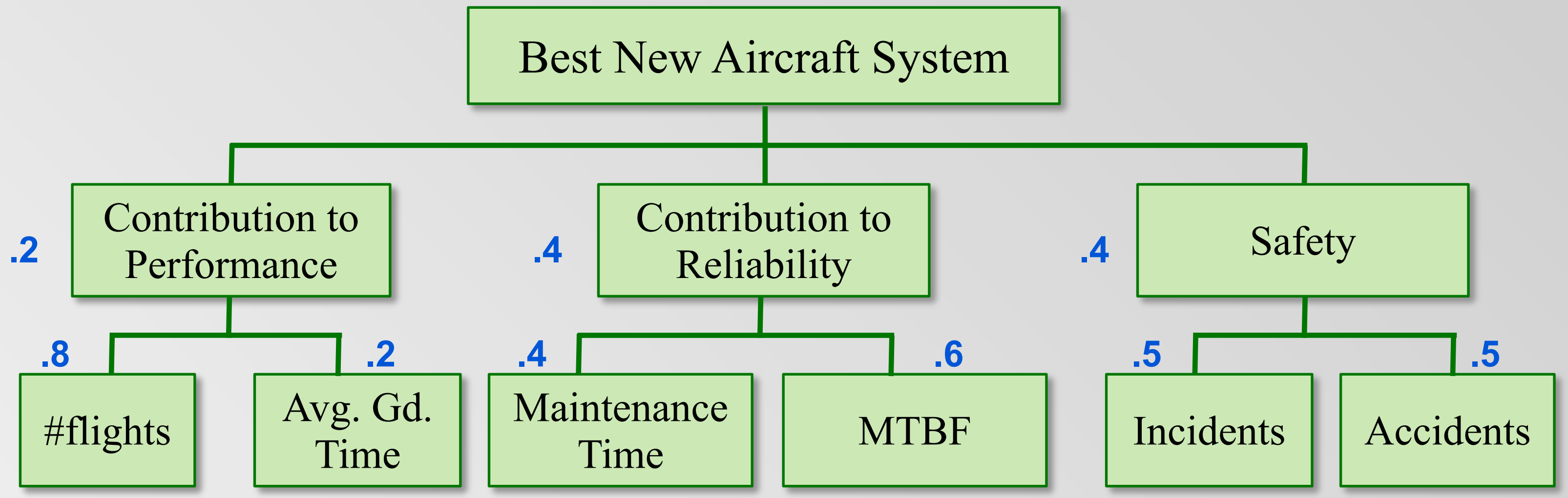
Scaled Scores for Flights Per Month of Operation

S_{f2} = 0 (since it is the worst) S_{f3} = 1 (since it is the most preferred)

S_{f1} = (x-worst)/(best-worst) = (385 – 420)/(350 - 420) = 0.5

Example Problem – Choose the Best Aircraft

Value Hierarchy With Weights



Bottom Row Weights .16 .04 .16 .24 .20 .20

Simulation Results

Alternative Aircraft System	# flights per Op. Month	Avg. Ground Time (m)	Maintenance Time (days)	MTBF (days)	# of Incidents /100Kh	# of Accidents / 100Kh
1	0.5	1	0.33	0	0.6	1
2	1	0.5	1	0.2	0	0.25
3	0	0	0	1	1	0

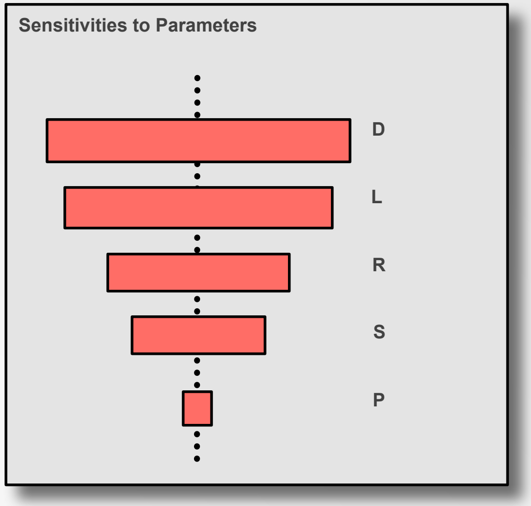
S1 = (.16)(0.5)+(.04)(1)+(.16)(.33)+(.24)(0)+(.2)(.6)+(.2)(1) **= 0.49**
S2 = .44
S3 = .44

Sensitivity Analysis

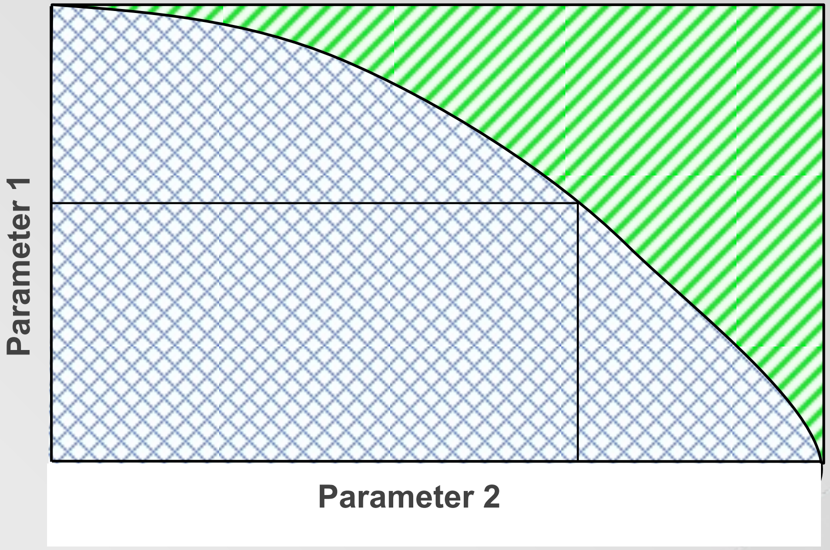
- Sensitivity analysis means varying the inputs to a model to see how the results change
- Sensitivity analysis is a very important component of exploratory use of models
  - model is not regarded as “correct”
  - sensitivity analysis helps user explore implications of alternate assumptions
  - human computer interface for sensitivity analysis is difficult to design well
- In many models we need to make assumptions we cannot test
  - Sensitivity analysis examines dependence of results on these assumptions

Visualizing Sensitivity Analysis Results

- Tornado Diagram
  - Visualizes result of varying a set of parameter through specified ranges on an output of interest



- Strategy Region Graph
  - Visualizes changes in optimal strategy as 2 parameters are varied through a range



Things to Remember

- Attributes must be independent for additive model
- Consider the Range of Variation of the attributes when eliciting weights
- Show bottom row (effective) weights to decision maker to ensure they reflect his or her preferences
- Examine results to understand why model makes the recommendations it does
- Do sensitivity analysis!

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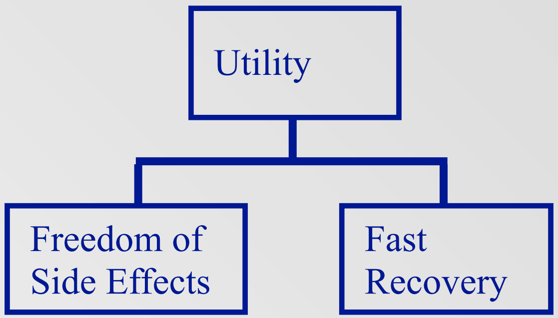
Dealing with Uncertainty

- Value Models: Multi-attribute Utility
- Decision Trees
  - A structured representation for options and outcomes
  - A computational architecture for solving for expected utility
  - Best with “asymmetric” problems (different actions lead to qualitatively different worlds)
- Influence Diagrams
  - A structured representation for options, outcomes and values
  - A computational architecture for solving for expected utility
  - Best with “symmetric” problems (different actions lead to worlds with qualitatively similar structure)
- Decision analysis software:
  - INFORMS software review page (http://www.informs.org/Apply-Operations-Research-and-Analytics/Software-Reviews)

Example: Patient Treatment

A patient is suspected of having a disease. Treated patients recover quickly from the illness, but the treatment has unplaceasant side effects. Untreated patients suffer a long and difficult illness but eventually recover.



Multi-Attribute Hierarchy

Goals:

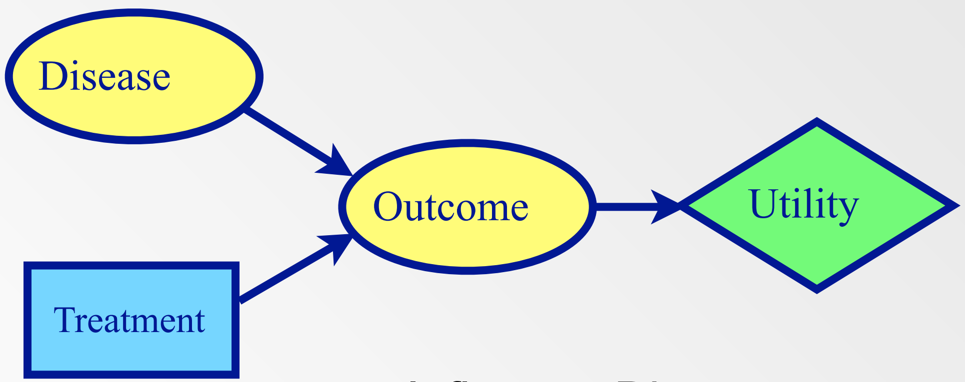
- Freedom from side effects
- Recovery

Options:

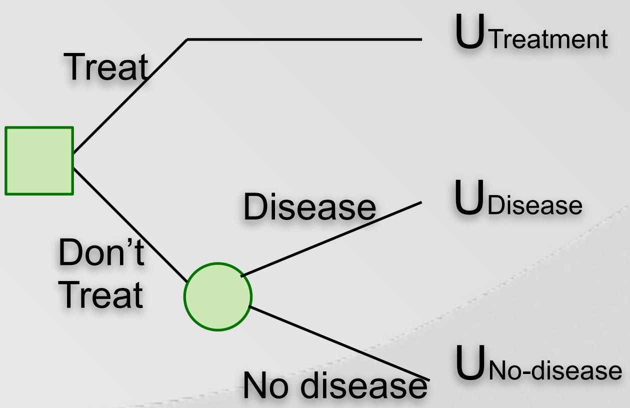
- Treat
- Don't treat

Outcomes:

- Sick/Well
- Side Effects / No Side Effects



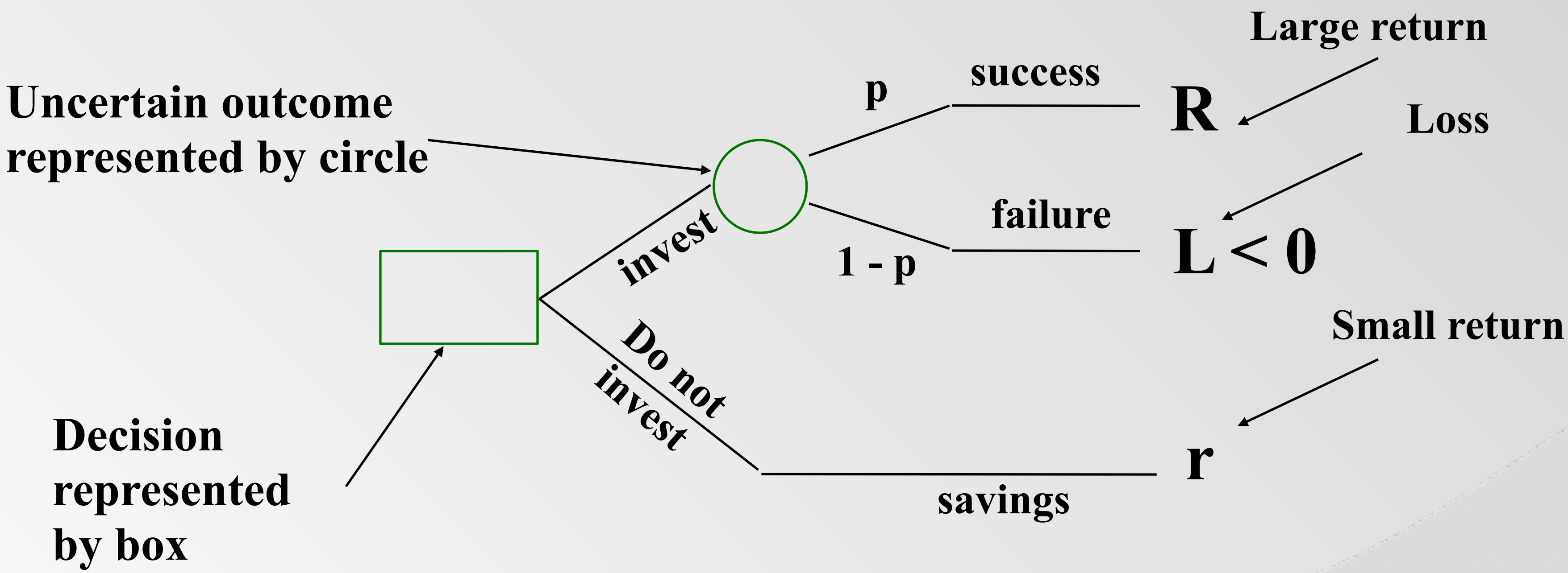
Influence Diagram



Decision Tree

Decision Trees

- Probabilities replace the weights
  - Account for uncertainty
  - Used to evaluate expected values
- Example - Venture Capital Problem

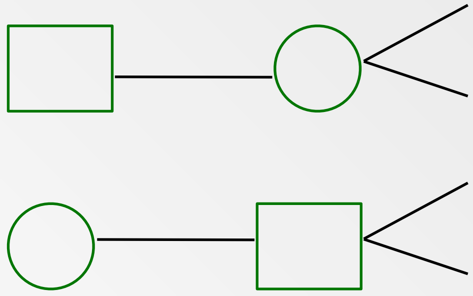


Decision Trees (continued)

- Expected return on investment:
  - If investment is made $E(I) = pR + (1 - p)L$
  - If investment not made $E(N) = r$
- Decision:
  - Invest if $pR + (1 - p)L > r$
  - Don't invest if $r > pR + (1 - p)L$

What would you do if $r = pR + (1 - p)L$?

- Decision Trees evaluated left to right



decision must be made before uncertain event takes place

decision is conditional on the known outcome of the uncertain event

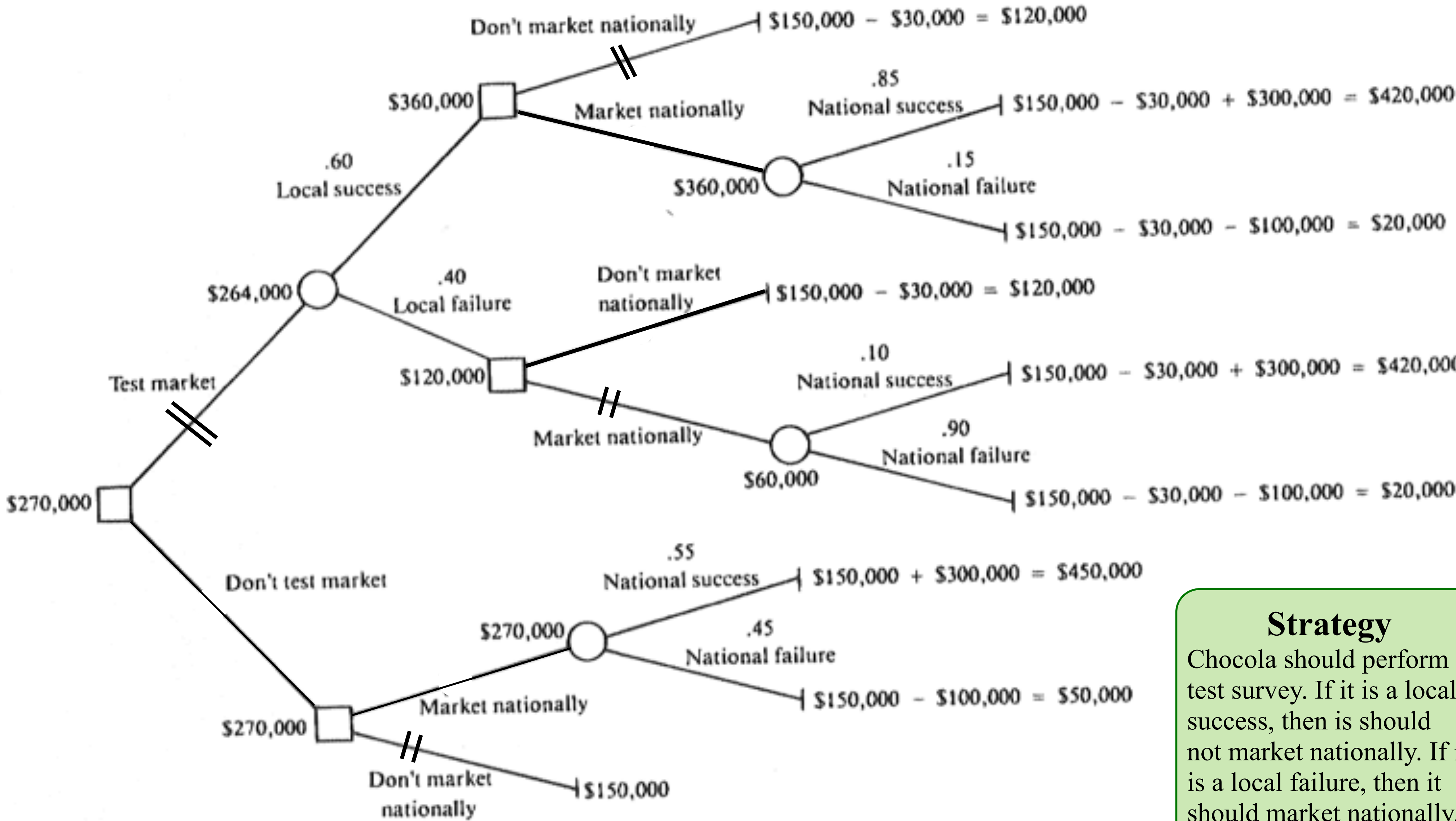
Chocola Decision Tree

- Colaco currently have $150,000 and wants to decide whether to market a new chocolate-flavored soda, Chocola.
- It believes that there is a chance of 55% of Chocola being a national success.
  - If it is, then Collaco's asset position will increase by $300,000.
  - If Chocola is a national failure, Colaco's asset position will decrease by $100,000.
- What should the company do?

Chocola Tree: Value of Information

- Before deciding whether to market Chocola, Colaco can perform a market survey at a cost of $30,000.
- There is a 60% chance that the study will yield favorable results (i.e. a local success), and 40% otherwise (i.e. a local failure).
  - If a local success is observed, there is an 85% that Chocola will be a national success.
  - If a local failure is observed, there is a 90% chance that Chocola will be a national failure.
- Should the company convey the market survey?
- What is the value of the information?

The Chocola Decision Tree



Strategy

Chocola should perform a test survey. If it is a local success, then is should not market nationally. If it is a local failure, then it should market nationally.

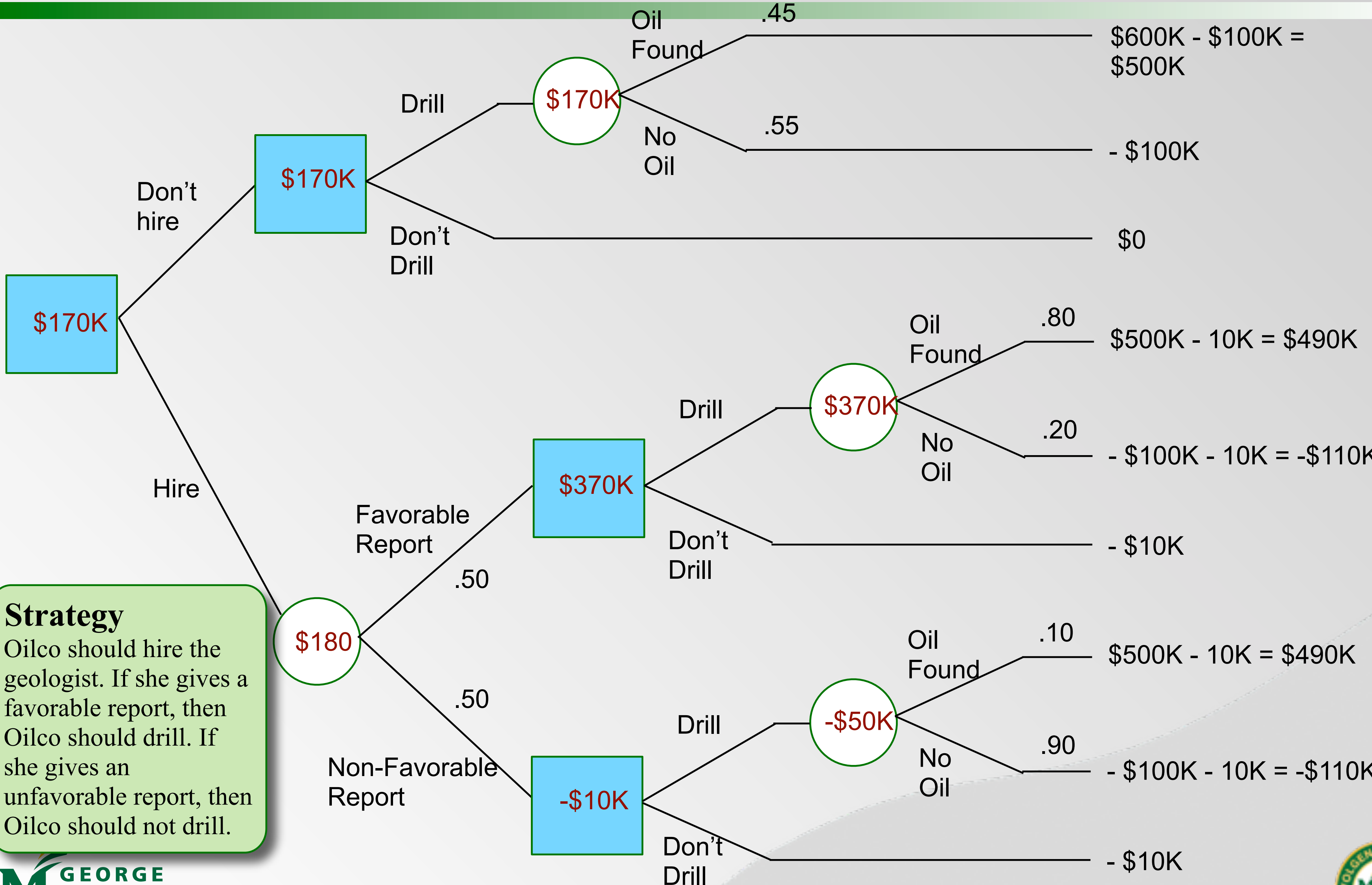
How Much the Test Should Cost?

- If no test is done, then the Expected Value for the decision tree is 270,000.
  - Let's call it EVWOI, for Expected Value With Original Information.
- If Colaco does a test, then the Expected Value for the decision tree becomes 294,000.
  - Let's call it EVWSI, for Expected Value With Sample Information.
- The difference is the Expected Value of Sample Information (EVSI), which is the difference between the EVWOI and the EVWSI.
  - Since it is $24,000 and the test costs $30,000, then Colaco should not perform the test.

Class Exercise: Oilco DT

- Oilco must determine whether or not to drill in the South China Sea.
  - It costs $100,000 and if oil is found the value is estimated to be $600,000.
  - At present, Oilco believes there is a 45% chance that the field contains oil.
- Before drilling, Oilco can hire (for $10,000) a geologist to obtain more information about the likelihood that the field contains oil.
- There is a 50% chance that the geologist will issue a favorable report and a 50% chance of an unfavorable report.
  - Given a favorable report, there is an 80% chance that the field contains oil.
  - Given an unfavorable report, there is a 10% chance that the field contains oil.
- Determine Oilco's optimal course of action.
- What is the EVSI?

Oilco Decision Tree



Strategy

Oilco should hire the geologist. If she gives a favorable report, then Oilco should drill. If she gives an unfavorable report, then Oilco should not drill.

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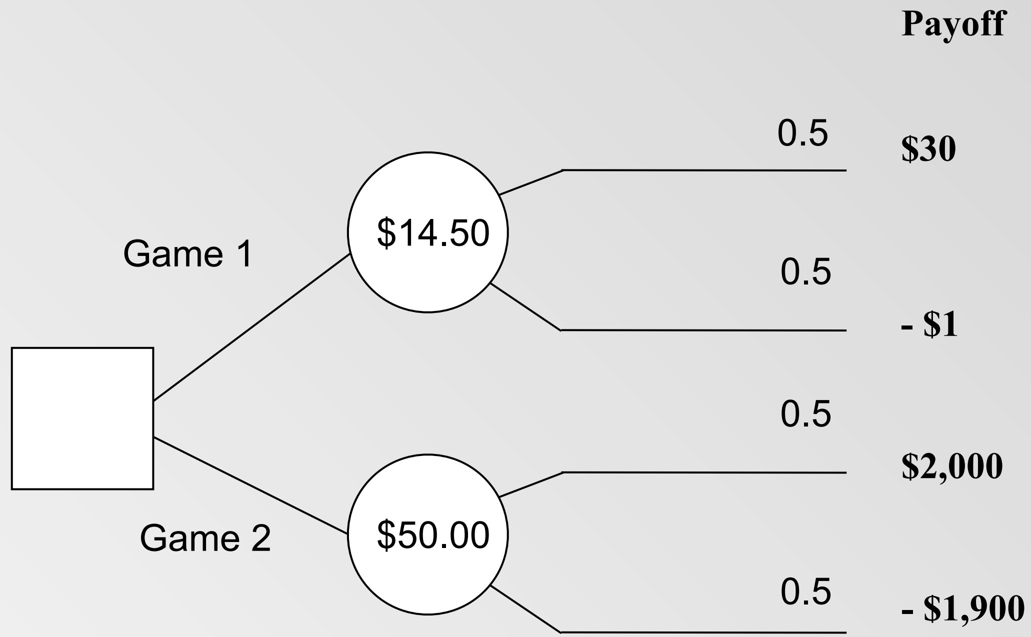
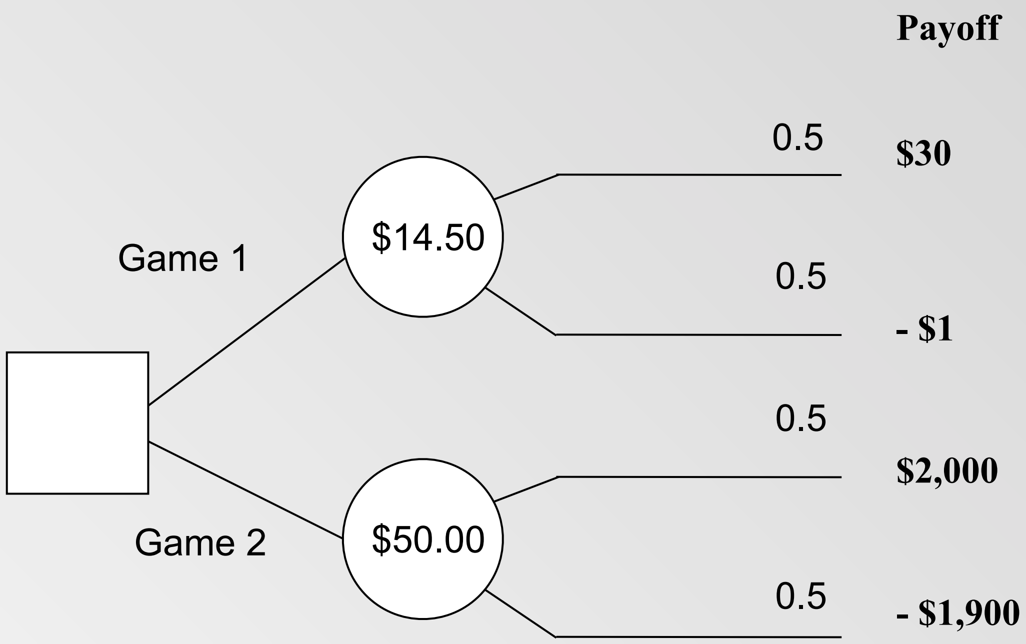
Risk - Introduction



Which game will you play?
Which game is risky?

Risk - Introduction



EMV
30*0.5+(-1)*0.5= 14.5

EMV
2000*.5+(-1900)*0.5= 50.0

Which game will you play?
Which game is risky?

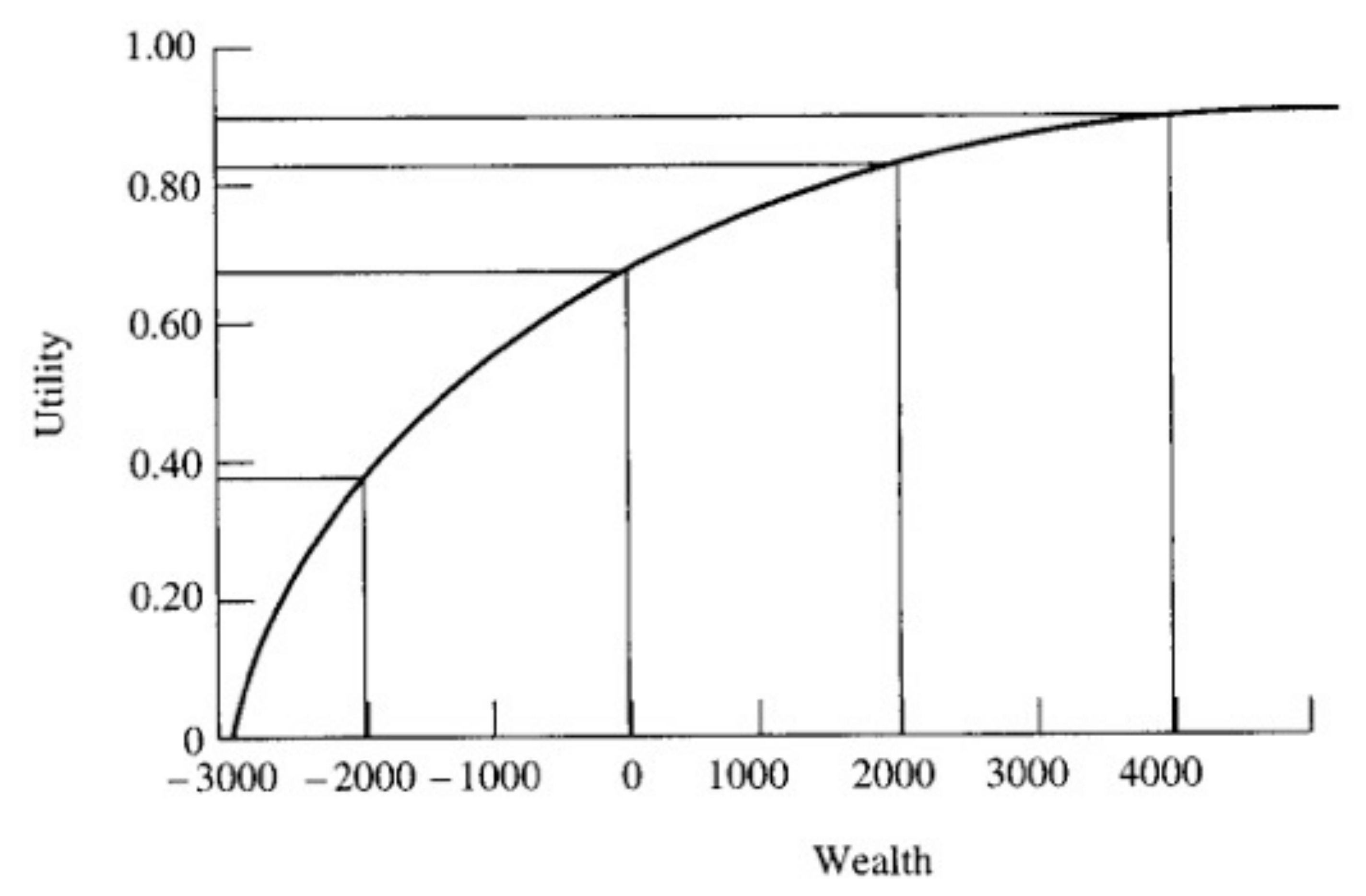
Going by expected monetary value (EMV) or the additive value function Game 2 has Higher EMV but also higher risk

CONCLUSION: EMV alone is not enough for decision making. Risk is very important too

What is an Utility function?

- A way to translate dollars into “utility units”
- It should help choose between alternatives by maximizing the expected utility
- Typical shapes of utility function include log, and exponential

Risk-Averse Utility Function

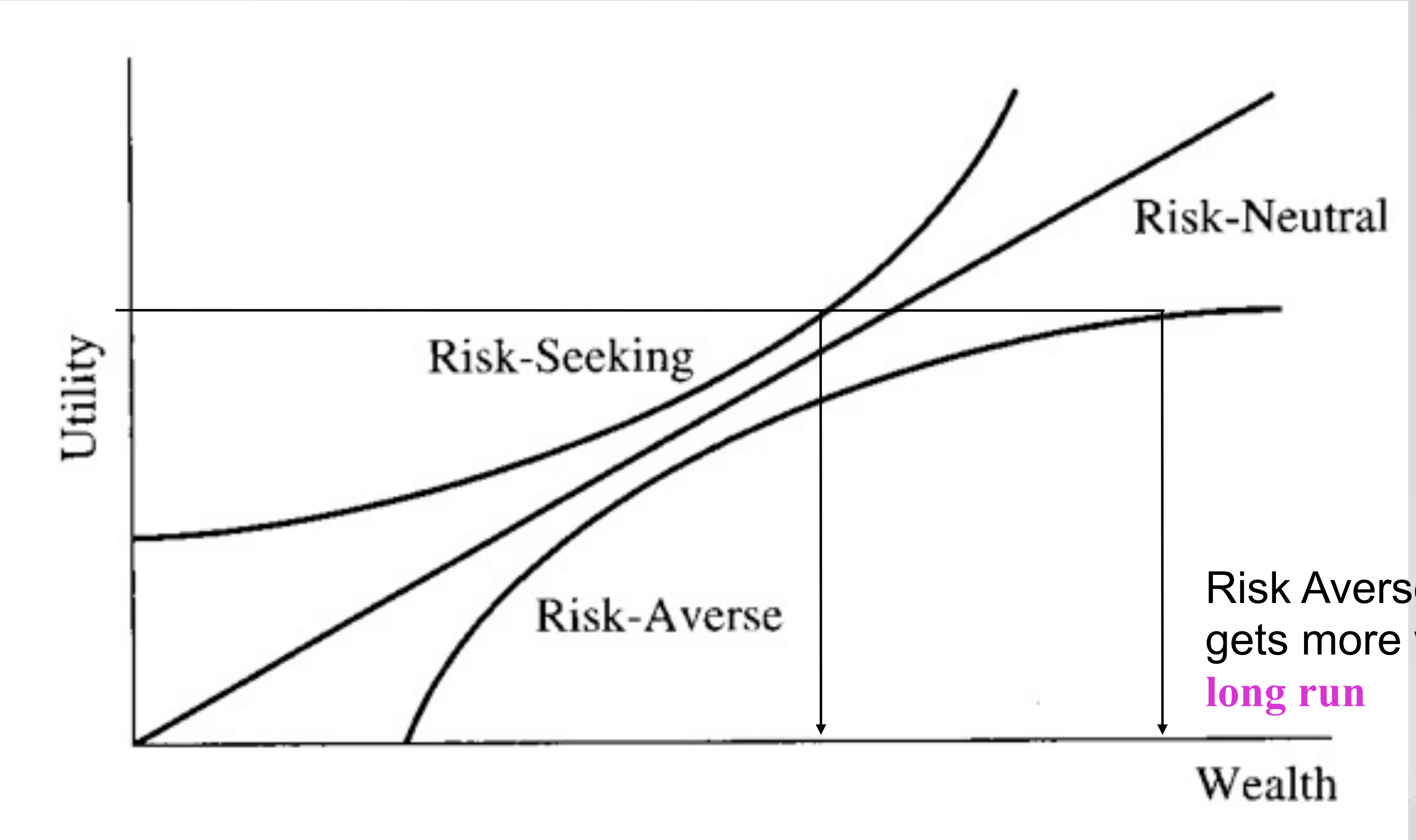


Note the Concave curve - this denotes Risk Averse - typical for most people

Risk averse person

- Imagine that you are gambling and you hit this situation:
  - Win $500 with prob 0.5 or lose $500 with prob 0.5
- A risk-seeking person will play the game but a risk averse person will try to trade in the gamble (try to leave the game) for a small penalty (example: pay $100 and quit).
- The EMV of the game is $0 and a risk averse person will trade in the gamble for an amount that is always less than the EMV value.
  - In this case -$100 <$0

Different Risk Attitudes

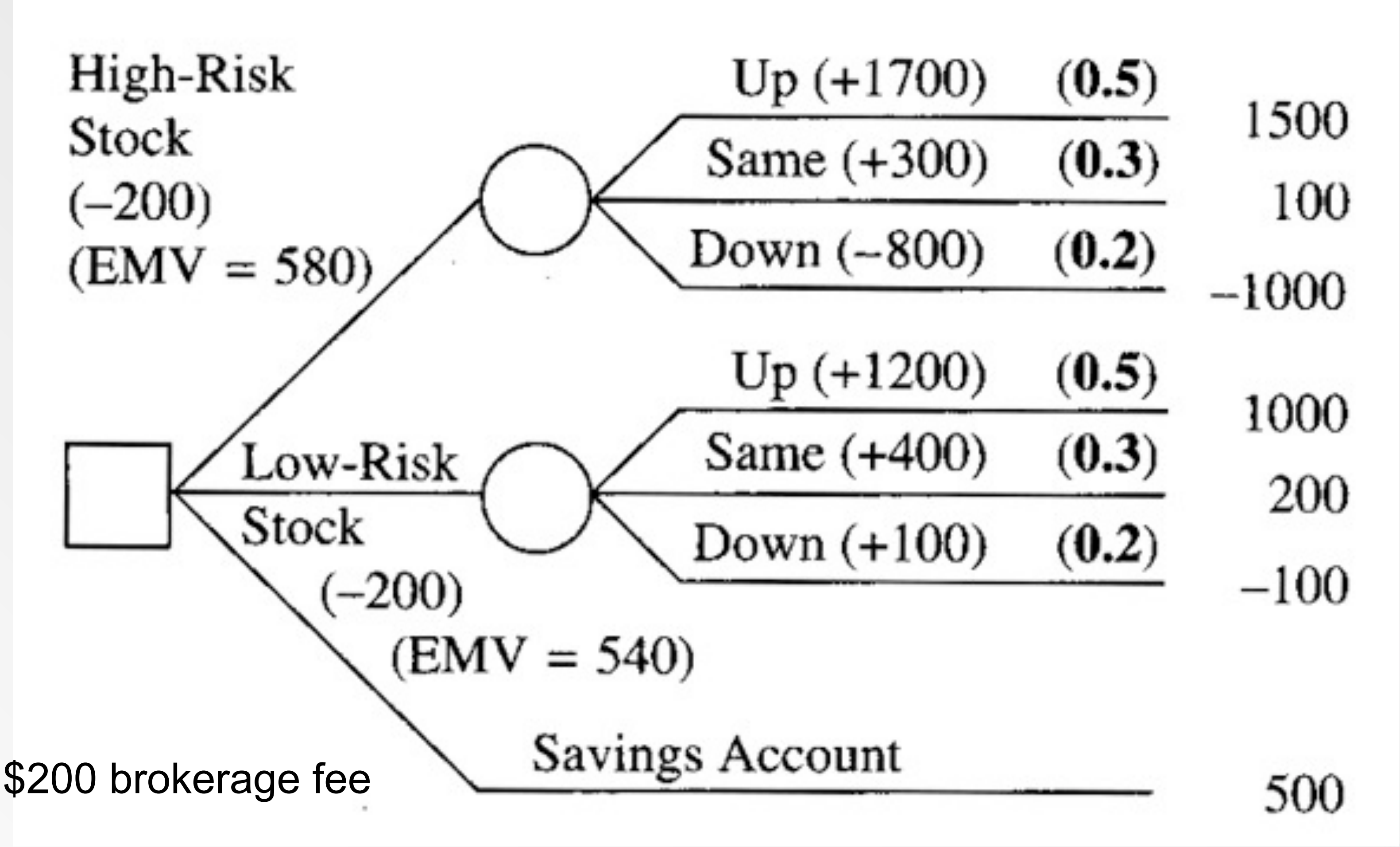


Ignores risk
Uses EMV

Risk Averse person gets more wealth **in the long run**

Different Risk Attitudes

Investing in the Stock Market

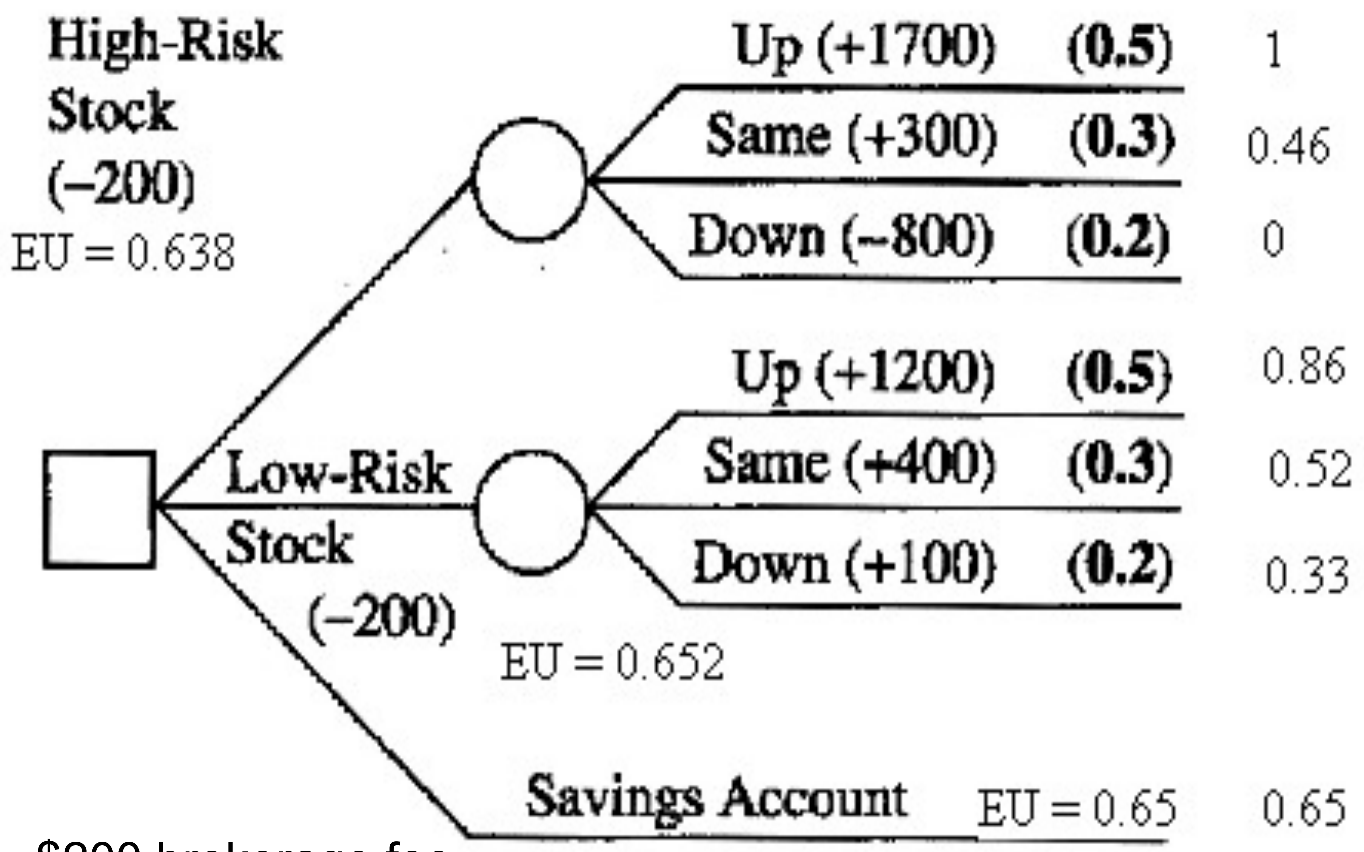


Solution to the decision tree is to invest in high-risk stock. Here risk is not incorporated

Utility function for investment

Dollar Value	Utility Value
1500	1.00
1000	0.86
500	0.65
200	0.52
100	0.46
-100	0.33
-1000	0.00

Investing in the Stock Market



0.5*1+0.3*0.46+
0.2*0=0.638

$200 brokerage fee

Solution to the decision tree is to invest in low-risk stock. Here risk is incorporated via utility function

Certainty Equivalents and Utility

- Suppose you face the following gamble:
  - Win $2000 with probability .5
  - Lose $20 with probability .5
- The expected monetary value for this game is:
  - $2000 * .5 + (-$20) * .5 = $990
- After some discussion, you decide to sell your position to an interested friend for $300.
  - That is, you "gave up" $690 ($990 - $300) just to avoid the risk involved in the gambling

In Other Words...

- A Certainty Equivalent is the amount of money you think is equal to a situation that involves risk.
- The Expected Monetary Value - EMV - is the expected value (in dollars) of the risky proposition
- A Risk Premium is defined as:
  - Risk Premium = EMV – Certainty Equivalent
- The Expected Utility (EU) of a risky proposition is equal to the expected value of the risks in terms of utilities, and EU(Risk) = Utility(Certainty Equivalent)

Some Interesting Facts...

- For risk-averse individuals (risk premium is positive), the horizontal EU line reaches the concave curve before the vertical EMV line.
- For risk-neutral individuals (risk premium is zero), the lines intercept at the linear 45 degree curve.
- For risk-seeking individuals (risk premium is negative), the horizontal EU line reaches the concave curve before the vertical EMV line.

Agenda

- The Decision Analysis Process
- Deciding with Values
- Qualitative and Quantitative Assessment

Text

- Modeling Uncertainty
- Risk Analysis