George Mason University
Department of Systems Enginnering and Operations Research
Examination for Waiver from SYST 500, 2002

1. Answer all questions
2. Exam time is 2 hours.
2. The exam is open book open notes.
3. Laptop or other computers are not allowed.
4. Show your work

1. 10 points. Determine whether the following vectors are linearly independent:

$$(1, 0, 1), \quad (2, -1, 0) \quad \text{and} \quad (0, 2, -4)$$

2. Consider the following system of equations:

$$\begin{aligned}
2x_1 + \ x_2 \qquad &= \quad 3 \\
-2x_1 \qquad + 2x_3 &= -2 \\
-4x_1 + \ x_2 + 7x_3 &= -2
\end{aligned}$$

   (a) Solve the problem via Gaussian elimination

   (b) Compute the determinant of the system matrix A.

3. Use Gaussian elimination to determine whether the following linear systems have a solution, and whether the solution is unique. When several solutions exist, find all solutions

   (a)
$$\begin{aligned}
-2x_1 + 8x_2 + 4x_3 &= 4 \\
x_1 - 4x_2 - 2x_3 &= 3
\end{aligned}$$

   (b)
$$\begin{aligned}
-2x_1 + 8x_2 + 4x_3 &= \ \ 4 \\
x_1 - 4x_2 - 2x_3 &= -2
\end{aligned}$$

4. Compute the eigenvalues and eigenvectors of the following matrix

$$A = \begin{pmatrix} -3 & 4 \\ 2 & -1 \end{pmatrix}$$

5. Compute the vector of first partial derivatives (the gradient) and the matrix of second partial derivatives (the Hessian) for the function

$$f(x) = e^{2x1+3x2} + 2x_1 x_2$$

6. Solve the following differential equations:

   (a)
$$y' + 3y = 0$$

(b)
$$y' + 3y = 9x$$

7.

(a) Use partial fraction decomposition to express the rational function below as the sum of rational functions with linear denominators.

$$\frac{4s}{s^2 + 2s - 3}$$

(b) Use the decomposition you obtained to determine

$$\int \frac{4s}{s^2 + 2s - 3} ds$$

(c) Use the decomposition you obtained to determine the inverse Laplace transform of

$$F(s) = \frac{4s}{s^2 + 2s - 3},$$

that is find the function $f(x)$ whose Laplace transform satisfies $L\{f(x)\} = F(s)$.

8. A jar contains 20 red jelly beans, 10 green jelly beans, and 5 black jelly beans. You stick your hand into the jar and scoop out a handful of 6 jelly beans.

(a) What is the probability that 3 of them will be black?

(b) What is the probability that your handful also includes some red jellybeans?